

Resonant nuclear transition in the $d\mu^6Li$ muonic molecule

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Abstract

The reaction rate of the nuclear fusion $d^6Li \rightarrow {}^8Be^*(2^+, 0)$ is estimated in the case where the nuclei are confined to each other by a muon. For the description of nuclear transitions, a method which is analogous to the Linear Combination of Atomic Orbitals has been used. Using the complex coordinate rotation method, we found that a molecular $d-\mu-{}^6Li$ state exists with energy $(-20.3084 - i0.0066)$ eV and $|\Psi_m(0)| = 0.44 \cdot 10^{-7} \text{ fm}^{-\frac{3}{2}}$. The nuclear wave functions needed, were constructed in the form of antisymmetrized products of harmonic-oscillator functions for the three-cluster approximation, $(dd\alpha)$, to the five-body $(NNNN\alpha)$ problem. It was found that the reaction rate λ is strongly dependent on the energy gap between the d^6Li threshold and the energy of the final ${}^8Be^*$ resonant state. The value of λ obtained by averaging over the width of this resonance, is $0.183 \cdot 10^{10} \text{ sec}^{-1}$.

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I. INTRODUCTION

Low-energy interactions among atomic nuclei are of great interest from pure theoretical as well as from practical point of view. From the former, they provide valuable information on the nucleon–nucleon interaction and on fine details concerning nuclear structure. By this we mean the manifestation of the main symmetries of strong interactions, such as charge symmetry, time invariance, etc at very low energies. Another issue here is the importance of the small components of nuclear wave functions. Very often these small components play a crucial role at super low energies [1], we are interested in. From the practical point of view, we mention here the importance of the low-energy nucleus-nucleus data in nuclear astrophysics.

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Nucleus-nucleus collision data are not available at energies below the keV region [2]. Meanwhile nuclear astrophysics deals with nuclear reactions occurring inside the stellar objects at these low energies and thus the available scattering data are usually extrapolated there. However, in certain systems of light nuclei, we may study nuclear reactions at very low energies, namely, reactions between nuclei confined inside muonic molecules. Being in a molecular state, nuclei have rather small relative kinetic energy and any possible reaction can be treated as a threshold reaction.

Nuclei in a molecule are relatively far from each other and the probability of their collision is very small. However, in some cases this is partly compensated by the large value of the nucleus-nucleus reaction cross-section which is due to the existence of a threshold resonance. In this paper we consider one such case, namely, the fusion of d and ${}^6\text{Li}$ nuclei, which forms the threshold resonant state ${}^8\text{Be}^*(2^+, 0)$.

Previously, we investigated three such cases, namely the $d\mu^7\text{Be}$, $p\mu^{10}\text{Be}$, and $t\mu^3\text{He}^*$ [3–5]. It was found that the reaction rates in all cases studied were rather high indicating that if such molecules are formed, the mixing of states could generate nuclear reactions at appreciable rate.

The paper is organized as follows: In Sec. II we apply an LCAO motivated approach to derive formal expressions for the reaction rate; in Sec. III we briefly describe how the three-body Coulomb problem $d - \mu - {}^6\text{Li}$ is treated; Sec. IV contains details concerning the employed nuclear wave functions. The potentials used and the procedure of calculating their matrix elements are described in Sec. V. Finally, our results are presented and discussed in Sec. VI.

II. REACTION RATE

Consider the molecular system $d - \mu - {}^6\text{Li}$ in its S -wave ground state. Using the complex coordinate rotation method (see forth Sec. III), we found that a quasistationary state exists with energy $(-20.3084 - i0.0066)$ eV which is in accordance with the early estimates of -19.8 eV [6] and -23.8 eV [7].

This binding energy is negligible on the nuclear energy scale, and beyond the typical accuracy with which the nuclear spectra are known. Thus, we can assume that the nuclear subsystem, $d{}^6\text{Li}$, of the above molecule, is close to its threshold energy. This threshold lies 22.2798 MeV above the ${}^8\text{Be}$ ground state i.e. almost at the same energy where the ${}^8\text{Be}$ nucleus has an excited state at $(22.2 - i0.8)\text{MeV}$ [8] with quantum numbers $(J^\pi, T) = (2^+, 0)$. This state can be formed also from the corresponding nuclear quantum numbers $d(1^+, 0)$, ${}^6\text{Li}(1^+, 0)$, and zero nucleus-nucleus angular momentum. Therefore, the nuclear subsystem $d{}^6\text{Li}$ of the molecule $d - \mu - {}^6\text{Li}$ can be fused via the reaction

$$d + {}^6\text{Li} \longrightarrow {}^8\text{Be}^* + 79.78 \text{ keV}. \quad (1)$$

To calculate the reaction rate of this process we employ a method which is motivated by the so-called Linear Combination of Atomic Orbitals (LCAO) [9] approach. We have used

this method previously to study the $d - \mu - {}^7\text{Be}$ [3] and $p - \mu - {}^{10}\text{Be}$ [4] systems.

Following the LCAO approach, we write the total Hamiltonian H , of the nine-body system considered here, in two equivalent forms corresponding to the initial and final clustering

$$H = H_1 + V^s, \quad (2)$$

$$H = H_2 + H^\mu, \quad (3)$$

where

$$H_1 = H_d + H_{6\text{Li}} + H_0 + V^c + H^\mu, \quad (4)$$

and

$$H_2 = H_{s\text{Be}}. \quad (5)$$

are the ‘unperturbed’ or ‘channel’ Hamiltonians. In these expressions, H_d , $H_{6\text{Li}}$, and $H_{s\text{Be}}$ are the nuclear hamiltonians of the clusters; H_0 is the kinetic energy operator describing the nucleus-nucleus relative motion while V^s is part of the strong potentials not included in H_d and $H_{6\text{Li}}$, i.e it represents interactions among nucleons belonging to different clusters; V^c is the nucleus-nucleus Coulomb potential and H^μ consists of the muon kinetic energy operator and its Coulomb interaction with the nuclear particles.

The ‘unperturbed’ Hamiltonians H_1 and H_2 have eigenstates $\Psi_1 = \mathcal{A}\{\Psi_d\Psi_{6\text{Li}}\Psi_{mol}\}$ and $\Psi_2 = \Psi_{s\text{Be}^*}\phi_\mu$, satisfying

$$H_i\Psi_i = E_i\Psi_i, \quad i = 1, 2 \quad (6)$$

and eigenenergies E_1 and E_2 which are very close to each other. In the above Ψ_{mol} is the three-body wave function of the molecular state; ϕ_μ is the atomic wave function of the muon in the Coulomb field of ${}^8\text{Be}$ and Ψ_d , $\Psi_{6\text{Li}}$, and $\Psi_{s\text{Be}^*}$ are the fully antisymmetric wave functions of the nuclear subsystems while \mathcal{A} is the antisymmetrizer involving nucleons from different nuclei.

In this system, the muon merely acts as a mediator creating an effective force which holds the nuclear clusters, d and ${}^6\text{Li}$, together at molecular distances. In order to gain a qualitative insight into the problem, one can assume that the effective interaction between d and ${}^6\text{Li}$ is as schematically shown in Fig. 1. When the clusters are close to each other they merge into ${}^8\text{Be}^*$ which can be considered as an eigenstate of the deep well. When they are separated at large distances, they can occupy the molecular eigenstate of the shallow well. The basic feature of this system is that the eigenvalues of these two eigenstates E_1 and E_2 are practically the same. As a result, the molecular and the compound nucleus states can be mixed. Despite the simplicity of such a two-cluster picture, we expect that the mixing of the two states can in reality take place as well.

The ‘perturbations’ V^s and H^μ mix the ‘unperturbed’ solutions Ψ_1 and Ψ_2 and shift the eigenenergies E_1 and E_2 . Thus, from the above arguments and following the LCAO approach, we look for a solution of the Schrödinger equation

$$H\Psi = E\Psi \quad (7)$$

in the form of a linear combination

$$\Psi = C_1\Psi_1 + C_2\Psi_2. \quad (8)$$

Using Eq. (8) in Eq. (7) and projecting with $\langle\Psi_i|$, we obtain the following algebraic linear equations for the coefficients C_i [4]

$$C_1(H_{11} - E) + C_2(H_{12} - EI) = 0, \quad (9)$$

$$C_1(H_{21} - EI^*) + C_2(H_{22} - E) = 0,$$

where $H_{ij} \equiv \langle\Psi_i|H|\Psi_j\rangle$ and $I = \langle\Psi_1|\Psi_2\rangle$, the overlapping integral. The determinant of the degenerated system (9) is equal to zero for the two values of the energy-parameter

$$E^{(\pm)} = \frac{1}{2(1 - |I|^2)} \left\{ H_{11} + H_{22} - (IH_{21} + I^*H_{12}) \pm \sqrt{[H_{11} + H_{22} - (IH_{21} + I^*H_{12})]^2 - 4(1 - |I|^2)(H_{11}H_{22} - H_{12}H_{21})} \right\}, \quad (10)$$

which are the ‘perturbed’ energy levels corresponding to the ‘unperturbed’ values E_1 and E_2 .

To find out which of the two levels $E^{(+)}$ and $E^{(-)}$ stems from the molecular and which from the nuclear state, we modify Eq. (10) by writing

$$H_{11} = E_1 + V_{11}^s, \quad (11)$$

$$H_{12} = E_1 I + V_{12}^s, \quad (12)$$

$$H_{21} = E_1 I^* + V_{21}^s, \quad (13)$$

$$H_{22} = E_2 + H_{22}^\mu, \quad (14)$$

where $V_{ij}^s = \langle\Psi_i|V^s|\Psi_j\rangle$ and $H_{22}^\mu = \langle\Psi_2|H^\mu|\Psi_2\rangle$. Noting that $(1 - |I|^2)^{-1} = 1 + g$, where $g = |I|^2 + |I|^4 + |I|^6 + \dots$ is a small quantity, we get

$$E^{(+)} = E_1 + \Delta_1, \quad (15)$$

$$E^{(-)} = E_2 + \Delta_2,$$

where

$$\Delta_1 = V_{11}^s + gH_{11} + (1 + g) \left[(\sqrt{1 + s} - 1)(E_1 - E_2 + V_{11}^s - H_{22}^\mu) - \text{Re}(IH_{21}) \right],$$

$$\Delta_2 = H_{22}^\mu + gH_{22} - (1 + g) \left[(\sqrt{1 + s} - 1)(E_1 - E_2 + V_{11}^s - H_{22}^\mu) + \text{Re}(IH_{21}) \right],$$

and

$$s = \frac{4}{(E_1 - E_2 + V_{11}^s - H_{22}^\mu)^2} \left[|I|^2 H_{11} H_{22} + |H_{21}|^2 - (H_{11} + H_{22}) \text{Re}(IH_{21}) + (\text{Im}(IH_{21}))^2 \right].$$

It is clear that $E^{(+)}$ is the ‘perturbed’ molecular energy level, and Δ_1 is the shift caused by the strong interaction between the nuclei, while Δ_2 is a negligible shift of the nuclear level caused by the muon cloud surrounding the ${}^8\text{Be}^*$ nucleus.

From the above expressions for Δ_1 and Δ_2 it is seen that our approach differs from the simple perturbation theory where $\Delta_1 \approx V_{11}^s$ and $\Delta_2 \approx H_{22}^\mu$. The LCAO wave function is, therefore, written

$$\Psi^{(+)} = C_1^{(+)}\Psi_1 + C_2^{(+)}\Psi_2, \quad (16)$$

where $C_1^{(+)}$ and $C_2^{(+)}$ are solutions of (9) corresponding to the energy $E = E^{(+)}$.

The probability P for the transition (1) is obtained as usual by the projection

$$P = \left| \langle \Psi_2 | \Psi^{(+)} \rangle \right|^2. \quad (17)$$

The P can be interpreted as the transition probability through the potential barrier. It is noted that the total wave function $\Psi^{(+)} \exp(-iE^{(+)}t/\hbar)$ oscillates at the barrier with frequency $\nu = |E^{(+)}|/2\pi\hbar$. Hence, in order to obtain the reaction rate λ , we must multiply the probability P by ν . Solving the algebraic system (9), we finally get

$$\lambda = \frac{|E^{(+)}|}{2\pi\hbar} \frac{|fI^* + 1|^2}{|f|^2 + fI^* + f^*I + 1}, \quad (18)$$

where

$$f = \frac{E^{(+)}I - H_{12}}{H_{11} - E^{(+)}} \quad (19)$$

Therefore to calculate the reaction rate we need the matrix elements V_{11}^s , V_{12}^s , V_{21}^s , H_{22}^μ , and the overlapping integral $I = \langle \Psi_1 | \Psi_2 \rangle$. This in turn requires the construction of the ‘unperturbed’ molecular ($d - \mu - {}^6\text{Li}$) and nuclear (${}^8\text{Be}^*$) wave functions Ψ_1 and Ψ_2 respectively.

III. MOLECULAR STATE

In the initial state, the nuclei d and ${}^6\text{Li}$ are very far apart and held by the muon at molecular distances (in the shallow well of Fig. 1). Since this separation is much greater than the nuclear sizes, we consider them as point-like charged particles interacting via a pure Coulomb potential. In order to solve the resulting three-body ($d\mu{}^6\text{Li}$) Coulomb problem, we employ the complex coordinate rotation method which has been successfully used in atomic physics for calculating positions and widths of resonant states (see, for example, Ref. [10] and references therein).

The main idea of this method consists in the complex rotation of the spatial coordinates

$$\vec{r} \rightarrow \vec{r} \exp(i\theta), \quad \theta = \theta_r + i\theta_i, \quad (20)$$

which makes the resonance state wave function square integrable. This, in turn, permits the use of variational methods for their location since the rotation leaves the resonance pole positions intact. Another advantage of this approach is that the search for resonances as eigenvalues in the complex plane has a rigorous mathematical foundation [13,14].

Muonic molecular systems have extremely narrow resonances, with widths which are difficult to calculate, because the accuracy of the numerical calculation for the complex energy has to be at least higher than the width magnitude. A variational method based on a random-tempered Slater-type exponential expansion exhibited high convergency and accuracy in calculations of muonic molecular bound states [11,12]. Here we demonstrate that this method is also successful in the case of $d\mu^6Li$ system.

The system of interest consists of three particles, a negative muon of mass m_μ and two nuclei of masses M_a and M_b , where a , hereafter, stands for a lithium nucleus and b for the deuteron. The Hamiltonian (in muonic atomic units $e = \hbar = m_\mu = 1$), after separating the center of mass motion, can be written as

$$H_{mol} = -\frac{1}{2m_a}\Delta_{\vec{r}_a} - \frac{1}{2m_b}\Delta_{\vec{r}_b} - \nabla_{\vec{r}_a} \cdot \nabla_{\vec{r}_b} - \frac{3}{r_a} - \frac{1}{r_b} + \frac{3}{R} \equiv T + V, \quad (21)$$

where \vec{r}_a and \vec{r}_b are the vectors towards the muon from the two nuclei, R denotes the distance between the nuclei, and $m_i = m_\mu M_i / (m_\mu + M_i)$ are the reduced masses of the respective muonic atoms ($i = a, b$). This set of coordinates, \vec{r}_a, \vec{r}_b , and \vec{R} , is more appropriate to handle the molecular three-body problem than the Jacobi coordinates (\vec{R}, \vec{r}_μ) commonly used in few-body problems.

In the complex coordinate rotation theory the resonant state is defined as the solution of the eigenvalue problem

$$(H_{mol}(\theta) - E)\Psi_{mol} = 0, \quad H_{mol}(\theta) = U(\theta)H_{mol}U^{-1}(\theta), \quad (22)$$

for the Hamiltonian $H_{mol}(\theta)$ analytically continued into the complex plane of dilation parameter θ of the transformation (20). The corresponding transformation of the molecular states in the Hilbert space is defined via

$$[U(\theta)F](\vec{r}) = e^{3\theta/2}F(e^\theta\vec{r}). \quad (23)$$

Such a transformation has a great computational advantage for systems with Coulomb interactions. The kinetic and potential parts scale as $\exp(-2\theta)$ and $\exp(-\theta)$, respectively, and the Hamiltonian can be written as

$$H_{mol} = Te^{-2\theta} + Ve^{-\theta}. \quad (24)$$

The variational wave function for the state with total orbital angular momentum $L = 0$ is expanded in the form

$$\Psi_{mol}(\vec{R}, \vec{r}_\mu) = \sum_{i=1}^N c_i \exp(-\alpha_i r_a - \beta_i r_b - \gamma_i R), \quad (25)$$

where the vector \vec{r}_μ directed from the center of mass of the nuclear subsystem to the muon, is the linear combination

$$\vec{r}_\mu = \frac{M_a \vec{r}_a + M_b \vec{r}_b}{M_a + M_b}$$

of \vec{r}_a and \vec{r}_b . The nonlinear parameters are chosen by a pseudorandom algorithm, previously used by Thakkar and Smith [15], and are generated by the formulae

$$\begin{aligned}\alpha_i &= A \langle \sqrt{2} [i(i+1)/2] \rangle, \\ \beta_i &= B \langle \sqrt{3} [i(i+1)/2] \rangle, \\ \gamma_i &= C \langle \sqrt{5} [i(i+1)/2] \rangle,\end{aligned}\tag{26}$$

where $\langle \dots \rangle$ denotes the fractional part of a number. The tempering parameters A, B, and C can be taken more or less arbitrarily. A reasonable requirement is that these parameters must be chosen to reflect the geometry of the molecule.

Once the nonlinear parameters α_i , β_i , and γ_i are fixed, the coefficients c_i can be found by solving the complex nonhermitian matrix eigenvalue problem

$$\mathbf{A} \mathbf{c}_k = E_k \mathbf{B} \mathbf{c}_k,\tag{27}$$

where

$$\mathbf{A} = \langle \Psi_{mol} | H_{mol}(\theta) | \Psi_{mol} \rangle, \quad \mathbf{B} = \langle \Psi_{mol} | \Psi_{mol} \rangle.$$

The corresponding discrete complex eigenvalues have the form

$$E_k = E_r - i\Gamma_m/2\tag{28}$$

where E_r gives the position and Γ_m the width of the resonance.

It should be emphasized that the exact eigenvalues do not depend on the rotating parameter θ . However, employing the ansatz (25), we deal with approximate eigenvalues which, therefore, have such dependence. On the other hand, in the area near the spectral points (including resonances) the dependence of eigenvalues on any parameter should be weak since the corresponding functional must be stable. Since the variational parameters of the expansion (25) are fixed by (26) and (27), then the only parameters which remain free are the real and imaginary parts of dilation angle θ . Therefore, the resonance positions and widths are deduced from the condition that a discrete complex eigenvalue is stabilized, i.e

$$\frac{\partial E_k}{\partial \theta} = 0,\tag{29}$$

with respect to variations of the complex dilation parameter θ .

To solve Eq. (27) numerically, the inverse iteration method adapted to symmetric complex matrices has been employed. This method is very effective, as far as computational effort is concerned, and stable to round-off errors. This enables us to extend the number of the expansion functions up to 1400 terms.

In our approach, the full three-body wave function Ψ_{mol} is not needed since in actual calculations of the fusion rate, only the effective two-body wave function $\Psi_m(\vec{R})$ describing the nucleus-nucleus relative motion is required. Moreover, Ψ_m will appear only in the integral expressions involving either the short-range strong potentials or the nuclear wave-functions localized within a volume extending to few fm. Hence, such integrals take into account only a small area around the $R = 0$ point. The boundary value $\Psi_m(0)$ is obtained from Ψ_{mol} by noting that after the complex rotation, the resonance wave function is square integrable and thus, the density $\rho_N^{(\theta)}(0)$ of the probability for the nuclei to be at vanishing internuclear separation, $R = 0$, is given by

$$\rho_N^{(\theta)}(0) = \frac{\int d\vec{r}_\mu \Psi_{mol}^\dagger(0, \vec{r}_\mu) \Psi_{mol}(0, \vec{r}_\mu)}{\langle \Psi_{mol} | \Psi_{mol} \rangle} . \quad (30)$$

where Ψ_{mol}^\dagger is the solution of the adjoint equation

$$\Psi_{mol}^\dagger H_{mol}(\theta) = \Psi_{mol}^\dagger E^* . \quad (31)$$

Finally, the rotated value of $\rho_N^{(\theta)}(0)$ can be easily transformed to $\theta = 0$ via

$$\rho_N^{(0)}(0) = e^{-3\theta} \rho_N^{(\theta)}(0) . \quad (32)$$

Therefore, the boundary value $\Psi_m(0)$ is

$$|\Psi_m(0)| = \sqrt{\rho_N^{(0)}(0)} .$$

Our calculations were performed in a 32 digits arithmetics. The values of M_{6Li} , M_d and m_μ are (in units of electron mass) 10961.9, 3670.481 and 206.7686, respectively, while the values of the tempering parameters used are A=1.8, B=1.8, and C=2.8.

In Fig. 2 we present the results obtained for an eigenvalue as a function of the complex dilation parameter and with variational wave function (25) having a total number of terms $N = 900$. The curves correspond to different values of the parameter $\alpha = e^{\theta r}$. Each curve connects points on the rotational path for the fixed value of α and different values of the rotational angle $\theta_i = 0, 0.02, \dots, 0.2$ in steps of $\Delta\theta_i = 0.02$. It is clearly seen that the rotational paths form the stabilized eigenvalue of $(-20.3084 - i0.0066)\text{eV}$ at $\theta_i \approx 0.1$, the accuracy being of the order of 10^{-4}eV . The Γ_m corresponds to the predissociation rate of $1.810^{13}\text{sec}^{-1}$. The boundary value of the nucleus-nucleus wave-function obtained by the above procedure is

$$|\Psi_m(0)| = 0.44 \cdot 10^{-7} \text{ fm}^{-3/2} .$$

IV. NUCLEAR MODELS

In order to reduce the eight-body nuclear system, d^6Li , in the initial channel to manageable proportions, 6Li is considered to be a $(d\alpha)$ -bound state system. In this way we have in the initial state a three cluster system, namely, the $dd\alpha$ one. It is known that such a

two-body assumption for the ${}^6\text{Li}$ ground state, is sufficiently accurate [18]. This model is also consistent with the treatment of the final state of the transition $d{}^6\text{Li} \rightarrow {}^8\text{Be}^*$, since when ${}^8\text{Be}$ nucleus is excited at $\sim 20\text{MeV}$, then only one of the two α -clusters in it is destroyed [18] forming two deuterons. This is schematically depicted in Fig. 3 together with two sets of possible Jacobi coordinates.

The complexity of the problem, especially the required multidimensional integrals (up to dimension of 10), necessitated the use of bound state wave functions with simple radial dependence which, however, are widely employed in the literature. These wave functions describe reasonably well the nuclear sizes which are crucial in our model. We shall discuss them below in more details.

A. The deuteron wave function

This is taken to be of Gaussian form

$$\Psi_d(\vec{r}) = \frac{N_d}{\sqrt{4\pi}} \exp\left(-\frac{d}{2}r^2\right) \chi^s \eta^a, \quad (33)$$

with

$$d = \frac{3}{8\langle r_d^2 \rangle},$$

and

$$N_d^2 = \frac{4}{\sqrt{\pi}} d^{\frac{3}{2}}.$$

where χ^s and η^a are the spin symmetric and isospin antisymmetric functions. The mean square radius is fixed by $\langle r_d^2 \rangle = (1.956\text{ fm})^2$ [19].

B. The ${}^6\text{Li}$ wave function

This is constructed from the product of Ψ_d and the harmonic oscillator $2s$ -wave function, describing the motion of the α particle with respect to the center of mass of the deuteron. It can be shown that, in the six-nucleon oscillator model of ${}^6\text{Li}$ -ground state, the degrees of freedom associated with the variable \vec{r}_2 (see Fig.3), has two quanta of excitation [18]. This is a consequence of the Pauli principle. Therefore, by using the $2s$ -function we, indirectly, take into account effects of the Pauli principle despite the fact that the α -particle is treated as elementary. Therefore the following ansatz was used

$$\Psi_{{}^6\text{Li}}(\vec{r}_1, \vec{r}_2) = \Psi_d(\vec{r}_1) \phi_{2s}(\vec{r}_2), \quad (34)$$

with the $2s$ -function having the usual form

$$\phi_{2s}(\vec{r}_2) = \frac{1}{\sqrt{4\pi}} \frac{\sqrt{\frac{8}{3}}}{\pi^{\frac{1}{4}} a^{\frac{3}{2}}} \left(\frac{r_2^2}{a^2} - \frac{3}{2} \right) \exp\left(-\frac{r_2^2}{2a^2}\right). \quad (35)$$

The oscillator parameter a ,

$$a^2 = \frac{9}{14} \left(\langle r_{6Li}^2 \rangle - \langle r_d^2 \rangle \right),$$

is chosen to reproduce the experimental mean square radius of 6Li , $\langle r_{6Li}^2 \rangle = (2.54 \text{ fm})^2$ [18].

C. The ${}^8Be^*(2^+, 0)$ wave function

As mentioned earlier, the wave function of ${}^8Be^*(2^+, 0)$ state, is assumed to be that of the five-body system ($nnpp\alpha$). It is well known that the ground state of 8Be can be described reasonably well by the two α -cluster model [18]. Using the eight-body harmonic oscillator model, however, this ground state is a $(1s)^4(1p)^4$ -configuration having four quanta of excitation. On the other hand, in the $\alpha - \alpha$ model all four quanta must be attributed to the $\alpha - \alpha$ relative motion because each of the two α -particles is in the $(1s)^4$ -configuration. Hence, for the ground state of 8Be the function $\mathcal{A}\{\Psi_{\alpha_1}\Psi_{\alpha_2}\phi_{3s}\}$ is expected to be a good approximation. Here

$$\phi_{3s}(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{2\sqrt{30}}{\pi^{\frac{1}{4}} c^{\frac{3}{2}}} \left[\frac{1}{4} - \frac{1}{3} \left(\frac{r}{c} \right)^2 + \frac{1}{15} \left(\frac{r}{c} \right)^4 \right] \exp\left(-\frac{r^2}{2c^2}\right)$$

is the normalized harmonic-oscillator function having two nodes and describing the relative $\alpha - \alpha$ motion with four quanta of excitation. The oscillator parameter c for this function,

$$c^2 = \frac{8}{11} \left(\langle r_{8Be}^2 \rangle - \langle r_\alpha^2 \rangle \right),$$

is chosen to reproduce the experimental size $\langle r_{8Be}^2 \rangle = (2.39 \text{ fm})^2$ of Berillium nucleus [20]. For $\langle r_\alpha^2 \rangle$ we have used the value $(1.671 \text{ fm})^2$ [21].

At 22.2 MeV above the ground state, the whole excitation energy is used to break one of the α -clusters [18]. It is therefore natural to assume that the other α particle remains in the same $3s$ -state with respect to the center of mass of the destroyed cluster and bears the whole excitation. Consequently the excited α cluster should have quantum numbers corresponding to the $(2^+, 0)$ state. The positive parity is obtained if we have two quanta of excitation in the oscillator model. The simplest configuration in this case is the dd -system with relative motion wave function ϕ_{2s} . The oscillator parameter in this case is adjusted in order to get the correct value of the excitation energy.

Therefore, the wave function of the resonant state ${}^8Be^*(2^+, 0)$ is

$$\Psi_{8Be^*}(\vec{r}_1, \vec{r}_4, \vec{r}_5, \vec{r}_6) = \mathcal{N}_2 \phi_{3s}(\vec{r}_6) \mathcal{A}\{\Psi_d(\vec{r}_1) \Psi_d(\vec{r}_4) \phi_{2s}(\vec{r}_5) \xi_{(12)(34)}\}, \quad (36)$$

where

$$\xi_{(12)(34)} = \eta_{12}^a \eta_{34}^a \sum_{\sigma\sigma'} \langle 1\sigma 1\sigma' | 2, \sigma + \sigma' \rangle \chi_{12}^s(\sigma) \chi_{34}^s(\sigma') \quad (37)$$

is the four-nucleon spin-isospin wave function constructed from the antisymmetric isospin functions η_{ij}^a and the symmetric spin functions χ_{ij}^s of the ij -nucleon pair and \mathcal{N}_2 is the normalization constant; \mathcal{A} is the antisymmetrizer consisting of the following permutations

$$\mathcal{A} = (12)(34) - (32)(14) - (42)(31) - (13)(24) - (14)(32) + (34)(12). \quad (38)$$

It is known that the antisymmetrization significantly reduces the differences among the various cluster representations of nuclei [18]. Moreover, in the harmonic oscillator model all possible cluster representations, after being antisymmetrized, become equivalent. This gives us reasons to believe that our wave function for ${}^8\text{Be}^*$, constructed in the cluster representation $dd\alpha$, is a reliable one.

D. The final and initial states

The wave function $\Psi_{s\text{Be}^*}$ appearing in the final state Ψ_2 can be obtained from (36). After carrying out the normalization and antisymmetrization it is written as follows

$$\begin{aligned} \Psi_{s\text{Be}^*}(\vec{r}_1, \vec{r}_4, \vec{r}_5, \vec{r}_6) = & \left[3 - \frac{144}{\sqrt{2}} d^{\frac{3}{2}} b^3 \frac{(1 - 2db^2)^2}{(1 + 2db^2)^5} \right]^{-\frac{1}{2}} \\ & \phi_{3s}(\vec{r}_6) \left\{ \Psi_d(\vec{r}_1) \Psi_d(\vec{r}_4) \phi_{2s}(\vec{r}_5) \xi_{(12)(34)} \right. \\ & - \Psi_d\left(\frac{\vec{r}_1}{2} - \vec{r}_5 + \frac{\vec{r}_4}{2}\right) \Psi_d\left(\frac{\vec{r}_1}{2} + \vec{r}_5 + \frac{\vec{r}_4}{2}\right) \phi_{2s}\left(\frac{\vec{r}_1 - \vec{r}_4}{2}\right) \xi_{(32)(14)} \\ & \left. - \Psi_d\left(\frac{\vec{r}_1}{2} \vec{r}_5 - \frac{\vec{r}_4}{2}\right) \Psi_d\left(\frac{\vec{r}_1}{2} + \vec{r}_5 - \frac{\vec{r}_4}{2}\right) \phi_{2s}\left(\frac{\vec{r}_1 + \vec{r}_4}{2}\right) \xi_{(13)(24)} \right\}. \end{aligned}$$

The initial state wave function $\Psi_1 = \mathcal{A}\{\Psi_d\Psi_{6Li}\Psi_{mol}\}$ can be similarly antisymmetrized with the help of the operator given in Eq. (38). Thus, after some algebra, we obtain

$$\begin{aligned} \Psi_1(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_\mu) = & \mathcal{N}_1 \left\{ \Psi_d(\vec{r}_1) \Psi_d(\vec{r}_4) \left[\phi_{2s}(\vec{r}_2) \Psi_{mol}(\vec{r}_3, \vec{r}_\mu) + \phi_{2s}\left(\frac{\vec{r}_2}{3} - \vec{r}_3\right) \Psi_{mol}\left(-\frac{5\vec{r}_2}{6} - \frac{\vec{r}_3}{2}, \vec{r}_\mu\right) \right] \xi_{(12)(34)} \right. \\ & - \Psi_d\left(\frac{\vec{r}_1}{2} + \frac{2\vec{r}_2}{3} + \vec{r}_3 + \frac{\vec{r}_4}{2}\right) \Psi_d\left(\frac{\vec{r}_1}{2} - \frac{2\vec{r}_2}{3} - \vec{r}_3 + \frac{\vec{r}_4}{2}\right) \\ & \times \left[\phi_{2s}\left(-\frac{\vec{r}_1}{4} + \frac{2\vec{r}_2}{3} - \frac{\vec{r}_3}{2} + \frac{\vec{r}_4}{4}\right) \Psi_{mol}\left(-\frac{\vec{r}_1}{3} - \frac{4\vec{r}_2}{9} + \frac{\vec{r}_3}{3} + \frac{\vec{r}_4}{3}, \vec{r}_\mu\right) \right. \\ & + \phi_{2s}\left(\frac{\vec{r}_1}{4} + \frac{2\vec{r}_2}{3} - \frac{\vec{r}_3}{2} - \frac{\vec{r}_4}{4}\right) \Psi_{mol}\left(\frac{\vec{r}_1}{3} - \frac{4\vec{r}_2}{9} + \frac{\vec{r}_3}{3} - \frac{\vec{r}_4}{3}, \vec{r}_\mu\right) \left. \right] \xi_{(14)(32)} \\ & - \Psi_d\left(\frac{\vec{r}_1}{2} - \frac{2\vec{r}_2}{3} - \vec{r}_3 - \frac{\vec{r}_4}{2}\right) \Psi_d\left(-\frac{\vec{r}_1}{2} - \frac{2\vec{r}_2}{3} - \vec{r}_3 - \frac{\vec{r}_4}{2}\right) \\ & \times \left[\phi_{2s}\left(-\frac{\vec{r}_1}{4} + \frac{2\vec{r}_2}{3} - \frac{\vec{r}_3}{2} - \frac{\vec{r}_4}{4}\right) \Psi_{mol}\left(-\frac{\vec{r}_1}{3} - \frac{4\vec{r}_2}{9} + \frac{\vec{r}_3}{3} - \frac{\vec{r}_4}{3}, \vec{r}_\mu\right) \right. \\ & + \phi_{2s}\left(\frac{\vec{r}_1}{4} + \frac{2\vec{r}_2}{3} - \frac{\vec{r}_3}{2} + \frac{\vec{r}_4}{4}\right) \Psi_{mol}\left(\frac{\vec{r}_1}{3} - \frac{4\vec{r}_2}{9} + \frac{\vec{r}_3}{3} + \frac{\vec{r}_4}{3}, \vec{r}_\mu\right) \left. \right] \xi_{(13)(24)} \left. \right\}. \end{aligned}$$

The normalization constant \mathcal{N}_1 is obtained numerically.

V. POTENTIALS AND MATRIX ELEMENTS

For the description of the strong interactions among the particles of the five-body ($NNNN\alpha$) system we need the nucleon-nucleon, v_{mn}^{NN} , and nucleon- α , $v_{m\alpha}^{N\alpha}$, potentials ($m, n = 1, 2, 3, 4$).

Since Ψ_1 and Ψ_2 are constructed from S -wave components only, the total angular momentum $J = 2$ of the ($NNNN\alpha$) system can be formed from the spin-momenta of the four nucleons. This is possible when the spins are aligned, i.e. when the total spin of any NN -pair equals 1. Thus, as a nucleon-nucleon potential, we employ the Malfliet-Tjon triplet potential [22],

$$v^{NN}(r) = V_1 \frac{e^{-\alpha_1 r}}{r} - V_2 \frac{e^{-\alpha_2 r}}{r}, \quad (39)$$

with $V_1 = 1438.72 \text{ MeV fm}$, $V_2 = 626.885 \text{ MeV fm}$, $\alpha_1 = 3.11 \text{ fm}^{-1}$, and $\alpha_2 = 1.55 \text{ fm}^{-1}$.

For the $N\alpha$ potential, we choose the one proposed in Ref. [23]

$$v^{N\alpha}(r) = -V_0 \exp(-wr^2) \quad (40)$$

where $V_0 = 55.774 \text{ MeV}$ and $w = 0.292 \text{ fm}^{-2}$.

For the matrix elements we have

$$V_{ij}^s \equiv \langle \Psi_i | v_{13}^{NN} + v_{14}^{NN} + v_{23}^{NN} + v_{24}^{NN} + v_{3\alpha}^{N\alpha} + v_{4\alpha}^{N\alpha} | \Psi_j \rangle. \quad (41)$$

Due to the symmetry properties of Ψ_1 and Ψ_2 with respect to nucleon permutations, we may write

$$V_{ij}^s = 4\langle \Psi_i | v_{14}^{NN} | \Psi_j \rangle + 2\langle \Psi_i | v_{4\alpha}^{N\alpha} | \Psi_j \rangle. \quad (42)$$

After antisymmetrization, the wave functions Ψ_1 and Ψ_2 acquire components with nonzero orbital angular momenta. However, in the integrals (42) the S -wave potential operators which implicitly include the projection operators $\mathcal{P}(\ell_{14} = 0)$ and $\mathcal{P}(\ell_{4\alpha} = 0)$ which retain only S -waves along the vectors

$$\begin{aligned} \vec{x} &= \frac{\vec{r}_1}{2} + \frac{2\vec{r}_2}{3} + \vec{r}_3 + \frac{\vec{r}_4}{2}, \\ \vec{y} &= -\frac{\vec{r}_2}{3} + \vec{r}_3 + \frac{\vec{r}_4}{2}, \end{aligned} \quad (43)$$

towards the first nucleon and the α -particle from the position of the fourth nucleon respectively (see Fig. 3). In order to perform this S -wave projection we make the additional integration

$$\begin{aligned} \mathcal{P}(\ell_{14} = 0)\Psi_i &= \frac{1}{4\pi} \int d\hat{\mathbf{x}} \Psi_i \equiv \Phi_{ix}, \\ \mathcal{P}(\ell_{4\alpha} = 0)\Psi_i &= \frac{1}{4\pi} \int d\hat{\mathbf{y}} \Psi_i \equiv \Phi_{iy}. \end{aligned}$$

Then the matrix elements are calculated as follows

$$\langle \Psi_i | v_{14}^{NN} | \Psi_j \rangle = (4\pi)^2 \int_0^\infty dr_1 dx (r_1 x)^2 \int d\vec{r}_2 d\vec{r}_4 d\vec{r}_\mu \Phi_{ix}^* v_{14}^{NN} \Phi_{jx}, \quad (44)$$

$$\langle \Psi_i | v_{4\alpha}^{N\alpha} | \Psi_j \rangle = (4\pi)^2 \int_0^\infty dr_1 dy (r_1 y)^2 \int d\vec{r}_2 d\vec{r}_4 d\vec{r}_\mu \Phi_{iy}^* v_{4\alpha}^{N\alpha} \Phi_{jy}. \quad (45)$$

For the overlapping integral I we have

$$I = 4\pi \int_0^\infty dr_1 r_1^2 \int d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_\mu \Psi_1^*(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_\mu) \Psi_{sBe^*}(\vec{r}_1, \vec{r}_4, \frac{2\vec{r}_2}{3} + \vec{r}_3, \frac{2\vec{r}_2}{3} - \frac{\vec{r}_3}{2}) \phi_\mu(\vec{r}_\mu), \quad (46)$$

where \vec{r}_1 is taken along the z -axis of the reference frame for the vectors \vec{r}_2 , \vec{r}_3 , and \vec{r}_4 . The dot products among these vectors, appearing in the integrand have the general structure

$$\vec{r}_m \cdot \vec{r}_n = r_m r_n [\sin \theta_m \sin \theta_n \cos(\varphi_m - \varphi_n) + \cos \theta_m \cos \theta_n],$$

where $m, n = 1, 2, 3, 4$ and θ_m, φ_m are the spherical angles of \vec{r}_m in that frame. The combination $\Psi_1^* \Psi_{sBe^*}$ contains dot products among the spin-isospin functions

$$\xi_1 \equiv \xi_{(12)(34)}, \quad \xi_2 \equiv \xi_{(14)(32)}, \quad \text{and} \quad \xi_3 \equiv \xi_{(13)(24)}.$$

It is easily seen that $\langle \xi_k | \xi_l \rangle = 1/2$ for all k and l .

The above multidimensional integrals (44), (45), and (46) involve an integration over the muon variable \vec{r}_μ (see Fig. 3). In order to reduce their dimensions, we exploit the fact that nuclear wave functions and potentials are localized within a small volume of the size which is not larger than few fm, while the molecular wave function $\Psi_{mol}(\vec{R}, \vec{r}_\mu)$ is practically constant when R is within a region of ~ 100 fm from the point $R = 0$.

Thus, in these integrals we can replace $\Psi_{mol}(\vec{R}, \vec{r}_\mu)$ by $\Psi_{mol}(0, \vec{r}_\mu)$. On the other hand, $\Psi_{mol}(0, \vec{r}_\mu)$ is the so-called united atom limit of the three-body molecular wave function and describes the motion of the muon in the joint Coulomb field of the two nuclei with zero separation between them. Hence, in this limit we have

$$\Psi_{mol}(\vec{R}, \vec{r}_\mu) \xrightarrow{R \rightarrow 0} \Psi_m(0) \phi_\mu(\vec{r}_\mu), \quad (47)$$

where $\phi_\mu(\vec{r}_\mu)$ is the atomic wave function, and $\Psi_m(0)$ is the limit ($R \rightarrow 0$) of the wave function describing the nucleus-nucleus relative motion inside the molecule (the calculated value of $\Psi_m(0)$ was given in Sec. III).

Therefore, by substituting the factorized expression (47) into the above integrals, the integration over the muon variable \vec{r}_μ is eliminated. The most formidable of the remaining integrals is one for the overlapping I which still has 10 dimensions.

Finally, we have to find the matrix element H_{22}^μ of the Hamiltonian $H^\mu(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)$ describing the muon motion in the Coulomb field generated by the nucleons. This Hamiltonian depends on the nucleon variables parametrically. Thus we have

$$H_{22}^\mu = \langle \Psi_{s_{Be^*}\phi_\mu} | H^\mu(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) | \Psi_{s_{Be^*}\phi_\mu} \rangle,$$

and again, exploiting the smallness of the nuclear size, we use the reliable approximation

$$\begin{aligned} H_{22}^\mu &\approx \langle \Psi_{s_{Be^*}\phi_\mu} | H^\mu(0, 0, 0, 0) | \Psi_{s_{Be^*}\phi_\mu} \rangle \\ &= \langle \phi_\mu | H^\mu(0, 0, 0, 0) | \phi_\mu \rangle. \end{aligned}$$

The last matrix element is just the eigenenergy of the muonic atom corresponding to the eigenstate $|\phi_\mu\rangle$ with the quantum numbers $(n\ell m)$. This eigenenergy is the one of a hydrogen-like atom and is expressed via the grand orbital quantum number n as $-(4/n)^2 Ry$ where $Ry = m_\mu e^4 / 2\hbar^2 = 2813.25 \text{ eV}$ is the Rydberg constant for muon.

A question then arises: which of the atomic states should be used here? The answer can be found with the help of the Born-Oppenheimer model. Indeed, in this model for the hydrogen-lithium system [17], only the so-called $3d\sigma$ -term provides the shallow well shown in Fig. 1. The solutions of the two-center Coulomb problem are usually classified by the quantum numbers $(n\ell m)$ of the united atom, ($R \rightarrow 0$), which is formed when the muon moves in the Coulomb field of the total charge of the nuclei. Thus, the symbol $3d\sigma$ means that, when the nuclei approach each other, the muon is in the atomic state with grand orbital quantum number $n = 3$, angular momentum $\ell = 2$, and $m = 0$. Moreover, when $R \rightarrow \infty$ the $3d\sigma$ -term correspond to free motion of lithium nucleus and $d\mu$ -atom in its ground state. Hence, as the zero point of the energy scale we should choose the ground state energy, $(-Ry)$, of $d\mu$ -atom. Therefore

$$H_{22}^\mu \approx -\left(\frac{4}{3}\right)^2 Ry + Ry = -\frac{7}{9} Ry.$$

We emphasize that the Born-Oppenheimer model was used here only to find the quantum numbers of the atoms which are formed in the limits $R \rightarrow 0$ and $R \rightarrow \infty$.

VI. RESULTS AND DISCUSSION

The numerical evaluation of the multidimensional integrals for the potential matrix elements and the overlapping integral, were performed with the help of the Haar-function expansion method [24]. The results thus obtained are

$$\begin{aligned} V_{11}^s &= -0.14524 \cdot 10^{-2} \text{ eV}, \\ V_{12}^s &= V_{21}^s = -6.2484 \text{ eV}, \\ H_{22}^\mu &= -5001.3 \text{ eV}, \\ I &= 0.42501 \cdot 10^{-6}. \end{aligned}$$

The use of these matrix elements in the formulae of Sec. II along with

$$E_1 = 22.2798 \text{ MeV} \quad \text{and} \quad E_2 = 22.2 \text{ MeV}$$

gives for the molecular level displacement Δ_1 , transition probability P , and reaction rate λ the following results:

$$\begin{aligned}\Delta_1 &= -0.963 \cdot 10^{-3} \text{ eV}, \\ P &= 0.606 \cdot 10^{-8}, \\ \lambda &= 0.298 \cdot 10^8 \text{ sec}^{-1}.\end{aligned}\tag{48}$$

At this point it is important to note that the compound nuclear state ${}^8\text{Be}^*(2^+, 0)$ is a resonant one with energy, $(22.2 - i0.8)$ MeV, having a rather large width $\Gamma = 1.6$ MeV. This width is much greater than the assumed difference,

$$\delta = E_1 - E_2 = 79800 \text{ eV},$$

between the molecular and compound nucleus levels. We further note that a resonance can be excited within a wide energy interval around the central resonant energy [25], and the relative probability of its excitation is defined by the Breit-Wigner factor

$$W(E) = \frac{\Gamma/2}{(E - E_2)^2 + \Gamma^2/4}.$$

Therefore, in order to obtain the correct values of Δ_1 , P , and λ , one should calculate them with different values of $\delta = E_1 - E_2$ and take the average value with $W(E)$ as the weight factor.

The dependence of Δ_1 , P , and λ on the energy level difference δ is shown in Table I. It is seen that the reaction rate λ significantly increases when $\delta \rightarrow 0$, and attains a very high value $0.246 \cdot 10^{16} \text{ sec}^{-1}$ when the levels E_1 and E_2 coincide. To include these contributions we note that the nuclear transition (1) can only take place if $E_1 \geq E_2$. Further, since δ cannot be greater than E_1 , the interval of δ for the averaging is $[0, E_1]$. Thus, the average value of the reaction rate is obtained via

$$\langle \lambda \rangle = \frac{\int_0^{E_1} d\delta \lambda(\delta) W(E_1 - \delta)}{\int_0^{E_1} d\delta W(E_1 - \delta)}.\tag{49}$$

The averages $\langle \Delta_1 \rangle$ and $\langle P \rangle$, can be similarly defined. The results which take into account the spreading of the final resonant state are

$$\begin{aligned}\langle \Delta_1 \rangle &= -0.116 \cdot 10^{-2} \text{ eV}, \\ \langle P \rangle &= 0.373 \cdot 10^{-6}, \\ \langle \lambda \rangle &= 0.183 \cdot 10^{10} \text{ sec}^{-1}.\end{aligned}$$

Our conclusions can be summarized as follows. We employed the LCAO-motivated approach to calculate the transition probability and reaction rate for the nuclear fusion inside the muonic $d - \mu - {}^6\text{Li}$ molecule by using the fact that ${}^8\text{Be}$ has an excited state near the $d {}^6\text{Li}$ threshold energy. The wave functions of the nuclear subsystems in these calculations

were constructed from antisymmetrized products of the harmonic oscillator functions in a three cluster ($dd\alpha$) approximation to the five-body model ($NNN\alpha$) while the three-body molecular problem was treated in the framework of the complex coordinate rotation method.

Within this model, the reaction rate strongly depends on the gap between the initial and final state energies. Since, however, the final state in our case is a resonance, we consider it reasonable to average the results over the permissible (by the reaction) energies covered by the resonance. The average results turned out to be two orders of magnitude higher as compared to those obtained using the center of the resonance. This makes the reaction (1) quite attractive for further theoretical as well as experimental investigations.

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TABLES

| δ (eV) | Δ_1 (eV) | P | λ (sec) ⁻¹ |
|-----------------|-------------------------------|------------------------------|-------------------------------|
| 0 | -6.25 | 0.499 | 0.246 10 ¹⁶ |
| 1 | 5.77 | 0.460 | 0.226 10 ¹⁶ |
| 10 | 3.00 | 0.188 | 0.921 10 ¹⁵ |
| 10 ² | 0.387 | 0.386 10 ⁻² | 0.190 10 ¹⁴ |
| 10 ³ | 0.376 10 ⁻¹ | 0.390 10 ⁻⁴ | 0.192 10 ¹² |
| 10 ⁴ | 0.245 10 ⁻² | 0.390 10 ⁻⁶ | 0.191 10 ¹⁰ |
| 79800 | -0.963 10⁻³ | 0.606 10⁻⁸ | 0.298 10⁸ |
| 10 ⁵ | -0.106 10 ⁻² | 0.385 10 ⁻⁸ | 0.189 10 ⁸ |
| 10 ⁶ | -0.141 10 ⁻² | 0.339 10 ⁻¹⁰ | 0.167 10 ⁶ |

TABLE I. The dependence of the molecular level displacement Δ_1 , transition probability P , and reaction rate λ on the energy difference δ between the molecular E_1 and compound-nucleus levels.

FIGURES

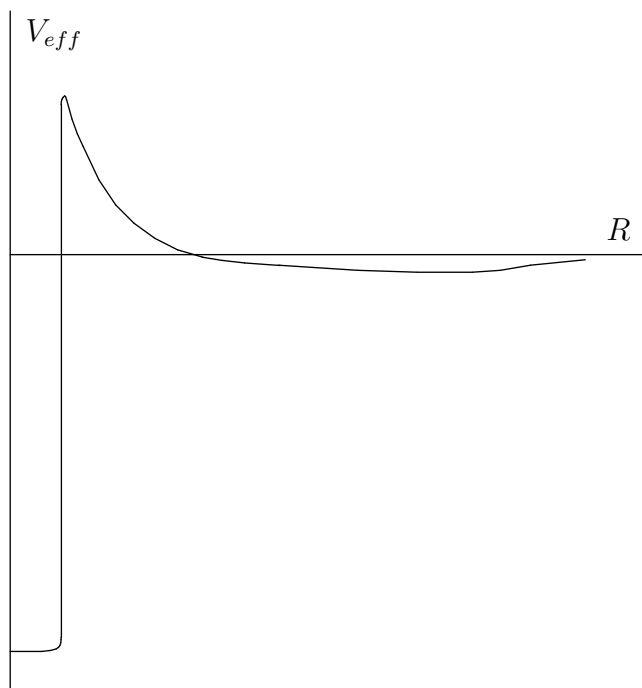


FIG. 1. Typical structure of the effective two-body interaction potential acting between the nuclei confined within a molecule.

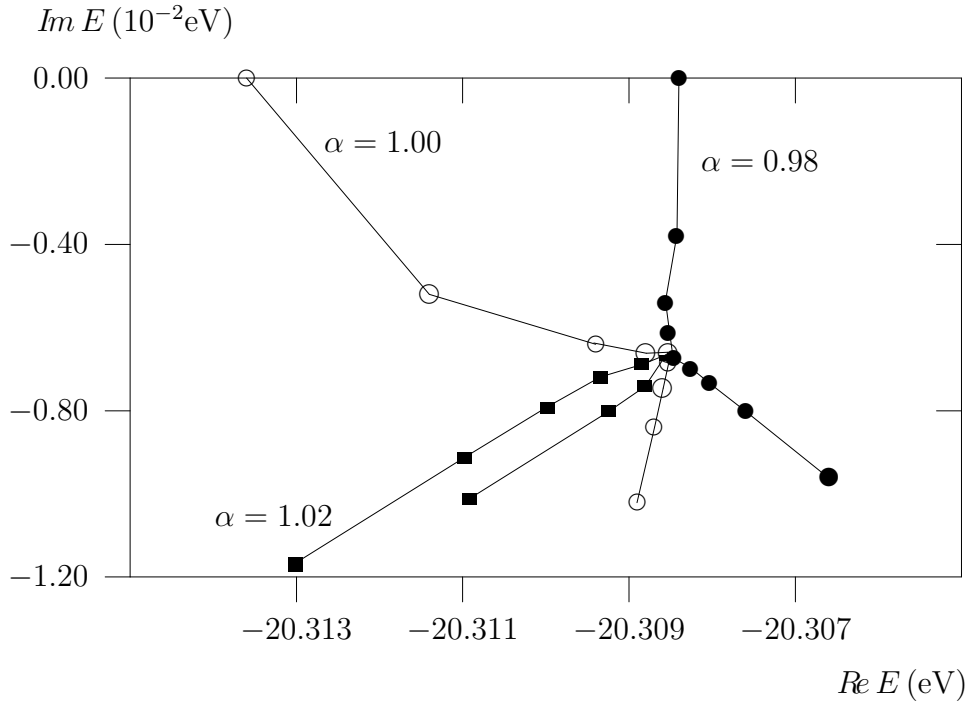


FIG. 2. Complex eigenvalue trajectories corresponding to variations of the rotation angle θ_i and for different values of dilatation parameter $\alpha = e^{\theta_r}$.

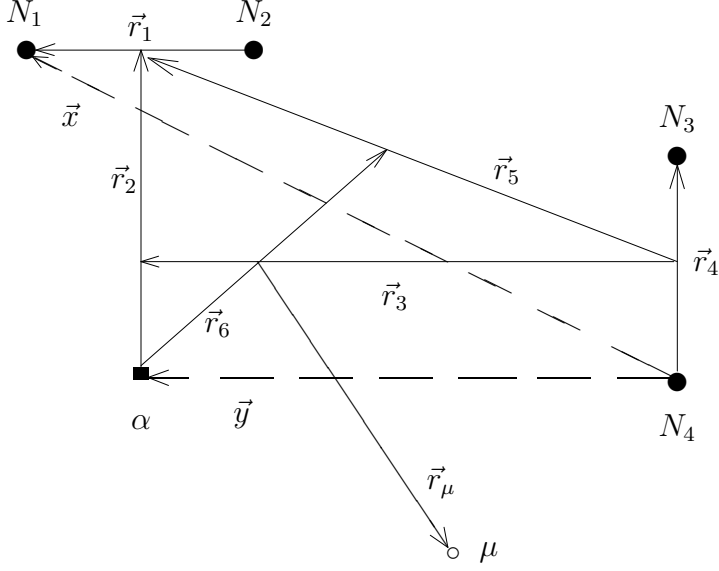


FIG. 3. Two alternative sets of Jacobi vectors $\{\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_\mu\}$ and $\{\vec{r}_1, \vec{r}_4, \vec{r}_5, \vec{r}_6, \vec{r}_\mu\}$ describing space configuration of the six-body system $(NNNN\alpha\mu)$. The \vec{x} and \vec{y} are auxiliary vectors defined by Eq. (43).